1 Summary

1. The **torque** of a motor is proportional to the **current** flowing through it ($\tau \propto i$).
2. The **angular velocity** of a motor is proportional to the **voltage** across it ($\omega \propto v$).

2 Torque/current derivation

The force upon a wire carrying current $\mathbf{i}$ in a magnetic field of strength $\mathbf{B}$ is $\mathbf{F} = \mathbf{i}l \times \mathbf{B}$, if l is the length of wire considered. It follows that the torque τ of a motor is proportional to the current i through the motor's coils: $\boxed{\tau \propto i}$.

3 Angular velocity/voltage derivation

The electrical power of a current i and voltage v is $P_e = iv$.

The mechanical work done by force over a given distance is $W_m = \int F\,dx$, so it follows that the work done by a torque over a given angle is $W_m = \int \tau\,d\theta$. The time-derivative of work is power, which in this case is $\frac{dW_m}{dt} = P_m = \frac{d}{dt}\left(\int \tau\,d\theta\right) = \tau\frac{d\theta}{dt} = \tau\omega$, where ω is the angular velocity.

Since energy must be conserved, power must be conserved as well. From that, it can be concluded that $P_e \propto P_m$, or $iv \propto \tau\omega$. Substituting i for τ (recall that those are proportional), $\tau v \propto \tau\omega \implies \boxed{\omega \propto v}$.